

## THE WEIGHTED AVERAGE COST OF CAPITAL

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### INTRODUCTION

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In the following all variables and parameters are stochastic variables, either in themselves or by being derived as functions of other stochastic parameters or variables (this even holds for the tax rate and tax regime). As such WACC is a stochastic variable defined by the variables and parameters below, but more important by the capital structure of the company and its value – which themselves are stochastic variables determined by the distributions for sale, prices, costs and investments.

We will use the mean values for all variables/parameters in the discussion and tables, but give the estimated probability distribution for WACC in the case used as example at the end.

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### WACC

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The S@R model puts emphasis on correct estimation of the weighted average cost of capital (WACC). The WACC is probably the single most important factor beside the return on invested capital (ROIC), when estimating company value – the basis for most strategy and performance evaluation methods. It is also the discount rate (time value of money) used to convert expected future cash flow into present value for all investors.

To be consistent with the Free Cash Flow or Economic Profit approach, the estimated cost of capital must comprise a weighted average of the marginal cost of all sources of capital -debt, equity etc that involves cash payment, now or in the future - excluding non-interest bearing liabilities (in simple form):

$$\text{WACC} = C_d(1-t) \cdot D/V + C_e \cdot E/V, \quad \text{where:}$$

$C_d$  = Pre-tax debt nominal interest rate,

$C_e$  = Opportunity cost of equity capital,

$t$  = Corporate marginal tax rate,

$D$  = Market value of interest bearing debt,

$E$  = Market value of equity,

$V$  = Market value of entity ( $V=D+E$ ).

The weights ( $D/V$  and  $E/V$ ) used in the calculation are the ratio between the market value of each type of debt and equity in the capital structure, and the market value of the company.

To estimate WACC we then need to:

1. Establish the market value weights for the capital structure,
2. Estimate the opportunity cost of non-equity financing,
3. Estimate the opportunity cost of equity financing.

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## THE MARKET VALUE WEIGHTS

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To establish the market value weights for the capital structure, we first need to establish the value of the company. But to find the company value we need WACC. There are two solutions to this circularity problem:

- A. To use a target capital structure, resulting in the use of one WACC for the entire forecast.
- B. To solve the implicit equation by numerical methods, and use a WACC that reflects the capital structure for the every year in the forecast.

Most approaches use the first solution, assuming that there is a target capital structure and that it is possible to achieve that target - in every period in the future. In real life this is usually not possible, there is retirement plans for all type of debt and often-unnecessary cost involved with early or late retirement. If anticipated changes in capital structure are expected to significantly affect the value of the company the only sound solution is to have a WACC calculated for every year. Most companies however, have a target capital structure at points of investment or when capital increases are necessary.

S@R has chosen to the last solution, by numerically solving the implicit equation. This implies that the market value weights are found for every type of capital in the capital structure for every future period analysed.

*“The theoretically correct approach to capital structure is to use a different WACC for each year that reflects the capital structure for the year” (Valuation, Tom Copeland et al)*

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## THE OPPORTUNITY COST OF EQUITY AND NON-EQUITY FINANCING

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To be consistent with the Free Cash Flow or Economic Profit approach (or other valuation methods), the estimated cost of capital must:

1. Use interest rates and cost of equity of new financing at current market rates - not at historical cost,

2. Be computed after corporate taxes,
3. Be adjusted for systematic risk born by each provider of capital,
4. Use nominal rates built from real rates and expected inflation.
5. When financing is done in foreign currency, we will also need to forecast future currency spot rates (and volatility).

*When financing is done in foreign currency, or with a portfolio of currencies, the steps above will have to be repeated for each currency (market). The volatility of the effective financing rate for each currency will then be “added” to the probability distribution for WACC. The volatility of the effective financing rate can then be studied and its effect on WACC and its distribution calculated.*

Limiting issues about country risk to the parameters affecting cost of capital, especially tax rates and tax regimes, this can be taken into account through the probability distribution for these parameter and the spot rates. A better approach would be to use “event trees” to estimate WACC under different scenarios.

With or without currency exposure the resulting company value and its probability distribution will reveal the exposure of different funding strategies. However we need to forecast the future risk free rates

## FORWARD INTEREST RATE ESTIMATION

From the yield curve for treasury obligations, implicit forward risk free interest rates can be calculated. If only one value is given for the first period, the yield curve is taken as horizontal, and all rates will be set to that value. If two or more values are given, the rest of the yield curve will be estimated by the regression:

$$\ln(1+R_t) = a+b*\ln(t),$$

Where  $R_t$  is the rate at period  $t$ , the implicit forward rates are then calculated by:

$$(1+r_{t,n-m})^{**}(n-m) = (1+R_n)^{**}n / (1+R_m)^{**}m \quad n > m,$$

where  $r_{t,n-m}$  are the forward rate at period  $t$  for the forward period  $(n-m)$ .

### Expected Risk Free Rate of Return (% pa)

	2002	2003	2004	2005	2006
<b>Expected Risk Free Rate of Return</b>	<b>6,9</b>	<b>6,5</b>	<b>6,4</b>	<b>6,3</b>	<b>6,2</b>

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## OPPORTUNITY COST OF EQUITY

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The equation for the cost of equity (pre investor tax), using the Capital Asset Pricing Model (CAPM) is:

$$C = R + M * \beta + L, \quad \text{where:}$$

$R$  = risk-free rate of return,

$\beta$  = the levered systematic risk of equity,

$M$  = market risk premium,

$L$  = liquidity premium.

If tax on dividend and interest income differs (ex. dividend payments are taxed on the company's hand and interest on investors hand) the risk-free rate and the market premium has to be adjusted, assuming tax rate  $-t_i$ , for interest income:

$$R = (1-t_i) * R \quad \text{and} \quad M = M + t_i * R., \quad \text{where:}$$

$t_i$  = Investor tax rate

$R$  = tax adjusted risk-free rate

$M$  = tax adjusted market premium

The tax adjusted CAP equation will then be:

$$C = R + M * \beta + L_p$$

The pre-tax cost of equity can then be computed as:

$$C_e(\text{pre-tax}) = \frac{R / (1-t_d) + \beta * M / (1-t_d) + L_p / (1-t_d)}{(1-t_d)} =$$

$$R / (1-t_d) + \beta * M / (1-t_d) + L_p / (1-t_d)$$

Where the first line applies for an investor with a tax rate of  $-t_d$ , on capital income, the second line for an investor where tax on dividend and interest differs.

The levered beta ( $\beta$ ) is found from the un-levered beta ( $\beta$ ) by :

$$\beta = (1 + (1 - t_d) * D/E) * \beta$$

*Default values are set to:*

Tax adjusted market premium = 5.5 % , the average for Oslo Stock exchange 1970-

Investor tax rate = 28.0 %

Liquidity premium = 0.0 % , ~ 2% for small companies and up to 4-5% for unquoted companies.

Un-levered beta = 0.5

In the following tables we have given an example on the calculations. The example is from “real life” and the tables’ gives expected values for the variables/parameters. The expected values for WACC in the Mont Carlo simulation, does however not necessarily have to be the equal to the calculated values in the table, due to the probability distributions of the variables. The business models have an expected debt-equity ratio (leverage) of one. In the simulation the following tables will be calculated for every run, and produce the basis for estimating the probability distributions for the derived variables (WACC, ROIC, FREE CASH FLOW ETC).

## COST OF EQUITY (%) per ANNUM

	2002	2003	2004	2005	2006
Expected Risk Free Rate of Return	6,9	6,5	6,4	6,3	6,2
Tax Adjustment of Risk-free Rate	(1,9)	(1,8)	(1,8)	(1,8)	(1,7)
Tax Adjusted Risk Free Rate	5,0	4,7	4,6	4,5	4,5
Market Risk Premium	5,5	5,5	5,5	5,5	5,5
Tax Adjustment of Market Premium	1,9	1,8	1,8	1,8	1,7
Tax Adjusted Market Premium	7,4	7,3	7,3	7,3	7,2
Adjustment for beta $\neq$ 1.	0,7	0,7	0,7	0,7	0,7
Adjusted Market Risk Premium	8,2	8,1	8,0	8,0	8,0
Liquidity Premium	2,0	2,0	2,0	2,0	2,0
Post Investor Tax Expected Return	15,1	14,7	14,6	14,5	14,4
Investor Tax on Return on Equity	0,0	0,0	0,0	0,0	0,0
<b>Pre Investor Tax Expected</b>	<b>15,1</b>	<b>14,7</b>	<b>14,6</b>	<b>14,5</b>	<b>14,4</b>

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## OPPORTUNITY COST OF DEBT

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It is assumed that the pre-tax Debt Interest Rate can be calculated using Risk Adjusted Return On Capital (RAROC) as follows:

$$\text{Debt Interest Rate} = R_f + L_C + L_L + L_A + L_{RP}, \quad \text{where:}$$

$R$  = Risk Free Interest Rate, as given by the Yield Curve.,

$L_C$  = Lenders Funding Cost (*typical 0.5%*),

$L_L$  = Lenders Average Expected Loss (*typical 1.5%*),

$L_A$  = Lenders Administration Cost (*typical 0.8%*),

$L_{RP}$  = Lenders Risk Premium (*typical 0.5%*).

The simulation model has as a parameter named "Lenders Cost", which consist of the sum ( $L_C + L_L + L_A + L_{RP}$ ), expected for this type of industry and company. *The default value is 3.3%*

### COST OF DEBT (%) per ANNUM

	2002	2003	2004	2005	2006
Forward Short-term Risk-free Rate	6,5	6,3	6,2	6,2	6,1
Lenders Cost	3,3	3,3	3,3	3,3	3,3
<b>Pre Tax Cost of Short-term Debt</b>	<b>9,8</b>	<b>9,6</b>	<b>9,5</b>	<b>9,5</b>	<b>9,4</b>
Tax Shield on Interest Payment	(2,7)	(2,7)	(2,7)	(2,7)	(2,6)
<b>Post Tax Cost of Short-term Debt</b>	<b>7,1</b>	<b>6,9</b>	<b>6,9</b>	<b>6,8</b>	<b>6,8</b>
Forward Long-term Risk-free Rate	6,3	6,2	6,1	6,1	6,1
Lenders Cost	3,3	3,3	3,3	3,3	3,3
<b>Pre Tax Cost of Long-term Debt</b>	<b>9,6</b>	<b>9,5</b>	<b>9,4</b>	<b>9,4</b>	<b>9,4</b>
Tax Shield on Interest Payment	(2,7)	(2,7)	(2,6)	(2,6)	(2,6)
<b>Post Tax Cost of Long-term Debt</b>	<b>6,9</b>	<b>6,8</b>	<b>6,8</b>	<b>6,8</b>	<b>6,7</b>

### SYSTEMATIC RISK OF DEBT - RECONCILIATION TO CAPM-MODEL

	2002	2003	2004	2005	2006
Systematic Risk of Short-term Debt	0,3	0,3	0,3	0,3	0,3
Tax Adjusted Risk Free Rate	5,0	4,7	4,5	4,6	4,5
Tax Adjusted Market Premium x beta	2,1	2,3	2,3	2,3	2,3
<b>Short-term Debt Cost Reconciliated</b>	<b>7,1</b>	<b>6,9</b>	<b>6,8</b>	<b>6,9</b>	<b>6,8</b>
Systematic Risk of Long-term Debt	0,3	0,3	0,3	0,3	0,3
Tax Adjusted Risk Free Rate	5,0	4,7	4,5	4,6	4,5
Tax Adjusted Market Premium x beta	2,0	2,2	2,3	2,2	2,3
<b>Long-term Debt Cost Reconciliated</b>	<b>6,9</b>	<b>6,8</b>	<b>6,8</b>	<b>6,8</b>	<b>6,7</b>

### MARKET VALUE OF DEBT AND EQUITY

	2002	2003	2004	2005	2006
Market Value of Short-term Debt	2,5	295,7	310,0	314,9	311,0
Market Value of Long-term Debt	568,7	607,7	641,9	666,2	681,3
Market Value of Equity	14.471,7	16.771,7	19.186,4	21.767,3	24.507,1
<b>Market Value of Entity</b>	<b>15.316,9</b>	<b>17.675,0</b>	<b>20.138,3</b>	<b>22.748,4</b>	<b>25.499,4</b>

### VALUE WEIGHTS IN WEIGHTED AVERAGE COST OF CAPITAL

	2002	2003	2004	2005	2006
Short-term Debt % of Adj. Market Value	1,8	1,7	1,5	1,4	1,2
Long-term Debt % of Adj. Market Value	3,7	3,4	3,2	2,9	2,7
Equity % of Adj. Market Value	94,5	94,9	95,3	95,7	96,1
Adj. Market Value of Total Capital	100,0	100,0	100,0	100,0	100,0

### ADJUSTMENTS OF $\beta$ FOR LEVERAGE

	2002	2003	2004	2005	2006
Leverage	0,751	0,858	0,926	0,883	0,849
Un-levered beta ( $\beta$ Equity)	1,100	1,100	1,100	1,100	1,100
Increase in $\beta$ due to Leverage	0,595	0,680	0,733	0,699	0,673
<b>Levered beta (<math>\beta</math> Equity)</b>	<b>1,695</b>	<b>1,780</b>	<b>1,833</b>	<b>1,799</b>	<b>1,773</b>

### EXPECTED RETURN ON EQUITY ADJUSTED FOR LEVERAGE - PERIODIC RATES

	2002	2003	2004	2005	2006
Tax Adjusted Risk Free Rate	5,0	4,7	4,6	4,5	4,5
Tax Adjusted Market Premium	7,4	7,3	7,3	7,3	7,2
Adjustment for $\beta \neq 1.0$	5,2	5,7	6,1	5,8	5,6
Adjusted Market Risk Premium	12,6	13,0	13,3	13,1	12,8
Liquidity Premium	2,0	2,0	2,0	2,0	2,0
Post Inv. Tax Exp. Return on Equity	19,6	19,7	19,9	19,6	19,3
Investor Tax on Return on Equity	0,0	0,0	0,0	0,0	0,0
<b>Pre Inv. Tax Exp. Return on Equity</b>	<b>19,6</b>	<b>19,7</b>	<b>19,9</b>	<b>19,6</b>	<b>19,3</b>



## WEIGHTED AVERAGE COST OF CAPITAL (%) Market value weights

	2002	2003	2004	2005	2006
Short-term Debt part of WACC	0,1	0,1	0,1	0,1	0,1
Long-term Debt part of WACC	0,3	0,2	0,2	0,2	0,2
Equity part of WACC	18,5	18,7	19,0	18,7	18,5
<b>WACC Rate (%)</b>	<b>18,9</b>	<b>19,1</b>	<b>19,3</b>	<b>19,0</b>	<b>18,8</b>

As can be seen from the table above, the rate varies slightly from year to year. The relative low volatility is mainly due to the low gearing in the forecast period.

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### MONTE CARLO SIMULATION

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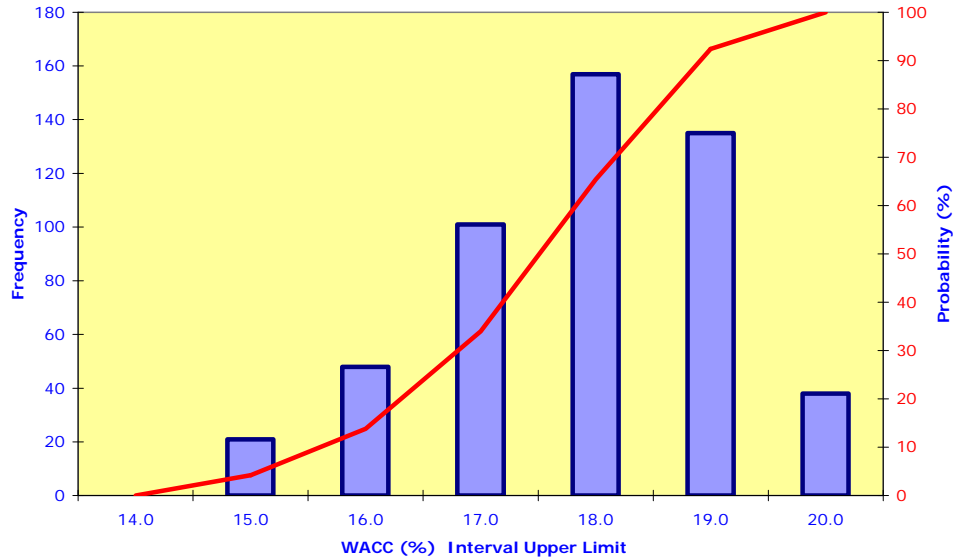
In the figure below we have shown the result from a Monte Carlo simulation (500 trials) of the company, and the resulting WACC for year 2002. This shows a much higher volatility than indicated in the table, and also that the expected value of WACC in this year is 17.4 %, compared with 18.9 % in the table. This indicates that the company will need more capital in the future, and that an increasing part will be financed by debt. A graph of the probability distributions for the yearly capital transactions (debt and equity) in the forecast period would have confirmed this.

*It is also important to note that the simulation was done assuming no volatility in the variables/parameters discussed above, only using the "normal risk" in running the company.*

For every year in the forecast period we will have a different WACC with its own probability distribution. In the forecast period WACC will increase/decrease depending on the company's development and the strategy chosen. The volatility will be a function of the distributions describing the technical and economic activities as well as the variables above.

Since WACC is the discount rate used to convert expected future cash flow into present value for all investors, an obvious goal will be to reduce the volatility as much as possible. The Coefficient of Variation can be used to measure the effect of different strategies on the WACC's volatility.

**Frequency and Probability Distribution for Expected Value WACC in Year 2002**  
 Expected Value 17.4, Standard deviation 1.19, Minimum 14.1, Maximum 19.5




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## RECONCILIATION OF WACC TO THE CAPM MODEL.

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As a test of the consistency in the calculations, the two following tables show the reconciliation of the calculated WACC to the CAPM model.

### SYSTEMATIC RISK OF TOTAL CAPITAL - Beta tot.

	2002	2003	2004	2005	2006
Risk of Short-term Debt	0,005	0,005	0,005	0,004	0,004
Risk of Long-term Debt	0,010	0,010	0,010	0,009	0,008
Levered Risk of Equity	1,602	1,689	1,747	1,722	1,704
<b>Risk of Total Capital</b>	<b>1,616</b>	<b>1,704</b>	<b>1,761</b>	<b>1,735</b>	<b>1,716</b>

# Strategy @Risk

## EXPECTED RETURN ON TOTAL CAPITAL

	2002	2003	2004	2005	2006
Tax Adjusted Risk Free Rate	5,0	4,7	4,6	4,5	4,5
Tax Adjusted Market Premium	7,4	7,3	7,3	7,3	7,2
Adjustment for $\beta \neq 1.0$	4,6	5,2	5,5	5,3	5,2
<b>Exp. Pre Tax Return on Total Capital</b>	<b>17,0</b>	<b>17,2</b>	<b>17,4</b>	<b>17,1</b>	<b>16,9</b>
Correction for Liquidity Premium	1,9	1,9	1,9	1,9	1,9
Correction for Tax Paid by Investor	0,0	0,0	0,0	0,0	0,0
<b><i>Total Capital Cost Reconciliated</i></b>	<b><i>18,9</i></b>	<b><i>19,1</i></b>	<b><i>19,3</i></b>	<b><i>19,0</i></b>	<b><i>18,8</i></b>

## VALUATION

The value of the company and the resulting value of equity can be calculated using either the Free Cash Flow or the Economic Profit approach. Correctly done, both give the same result with the use of this approach. This is the final test for consistency in the business model. The to calculations are given in the tables below, and calculated as the value at end of every year in the forecast period.

## FREE CASH FLOW VALUATION

	2002	2003	2004	2005	2006
Market Value of Free Cash Flow	7.219	8.034	8.636	9.060	9.238
Excess Marketable Securities	0	0	0	0	0
Continuing Value of Free Cash Flow	8.098	9.641	11.502	13.689	16.262
<b>Value of Entity by Free Cash Flow</b>	<b>15.317</b>	<b>17.675</b>	<b>20.138</b>	<b>22.748</b>	<b>25.499</b>
Value of Debt	(845)	(903)	(952)	(981)	(992)
<b>Value of Equity by Free Cash Flow</b>	<b>14.471</b>	<b>16.771</b>	<b>19.186</b>	<b>21.767</b>	<b>24.507</b>

# Strategy @Risk

## ECONOMIC PROFIT VALUATION

	2002	2003	2004	2005	2006
Invested Capital at Beg. of Period	0	254	388	514	655
Excess Marketable Securities	0	0	0	0	0
Capital Charge	0	49	75	98	123
NPV of Forecasted Economic Profit	7.512	8.080	8.590	8.944	9.049
Continuing Value of Economic Profit	7.805	9.292	11.086	13.193	15.673
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Value of Entity by Economic Profit	15.317	17.675	20.138	22.748	25.499
Value of Debt	(845)	(903)	(952)	(981)	(992)
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<b>Value of Equity by Economic</b>	<b>14.471</b>	<b>16.771</b>	<b>19.186</b>	<b>21.767</b>	<b>24.507</b>

*So we find that both methods using the same series of WACC, gives the same value for the company and the equity. This ensures that the calculations are both correct and consistent.*